Math 336 Exam 1 Solutions

Fall 2009 - Brad Hartlaub

1. Let U = U fills the order

V = V fills the order

W = W fills the order

M = Mistake

Given: $P(U) = .3$, $P(V) = .4$, $P(W) = .3$

$P(M|U) = .01$, $P(M|V) = .05$, and $P(M|W) = .03$

Find $P(U|M)$

Use Bayes Thm

$$P(U|M) = \frac{P(M|U)P(U)}{P(M|U)P(U) + P(M|V)P(V) + P(M|W)P(W)}$$

$$= \frac{.01(.30)}{.01(.3) + .05(.4) + .03(.3)}$$

$$= \frac{.003}{.032}$$

$$= .0938$$

2. a. The # of equally likely outcome sequences is $6^5 = 7,776$

The # of ways to get two pairs is

$$\underbrace{\binom{6}{2}}_{\substack{\text{\# of ways}\\\text{to select}\\\text{2 digits}}} \times \underbrace{\binom{5}{2}}_{\substack{\text{\# of ways}\\\text{to get}\\\text{first pair}}} \times \underbrace{\binom{3}{2}}_{\substack{\text{\# of ways}\\\text{to get}\\\text{2nd pair}}} \underbrace{\binom{4}{1}}_{\substack{\text{\# of ways}\\\text{to get}\\\text{last digit}}} = 1800$$

$$P(\text{two pairs}) = \frac{1800}{7776} = \frac{25}{108} = .2315$$

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#2 b. Find $P(\text{Full house})$

The # of ways to get 3 of a kind and a pair is:

$$\underbrace{\binom{6}{1}}_{\substack{\text{\# of ways}\\ \text{to select}\\ \text{1 digit}}} \times \underbrace{\binom{5}{3}}_{\substack{\text{\# of ways}\\ \text{to get 3}\\ \text{of a kind}}} \times \underbrace{\binom{5}{1}}_{\substack{\text{\# of ways}\\ \text{to get}\\ \text{2nd digit}}} \times \underbrace{\binom{2}{2}}_{\substack{\text{\# of}\\ \text{ways to}\\ \text{get a pair}}} = 300$$

$$P(\text{full house}) = \frac{300}{7776} = \frac{25}{648} = .0386$$

#3. $$P(C \mid A \cap B) = P(C \mid B) \underset{\substack{\text{def of}\\ \text{conditional prob}}}{\Rightarrow} \frac{P(A \cap B \cap C)}{P(A \cap B)} = \frac{P(B \cap C)}{P(B)}$$

$$\Rightarrow \frac{P(A \cap B \cap C)}{P(B \cap C)} = \frac{P(A \cap B)}{P(B)}$$

$$\underset{\substack{\text{def of}\\ \text{cond. prob.}}}{\Rightarrow} P(A \mid B \cap C) = P(A \mid B) \,//$$

#4. Let p = probability that an event will occur

$$\text{Odds} = \frac{p}{1-p}$$

Given: $$\frac{p}{1-p} = \frac{a}{b} \Rightarrow pb = a - ap$$

$$\Rightarrow p(a+b) = a$$

$$\Rightarrow p = \frac{a}{a+b} \,/$$

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5. $P(X = i) = \frac{i}{148}, \quad i = 40, 33, 25, 50$

Thus, $E[X] = 40 \cdot \frac{40}{148} + 33 \cdot \frac{33}{148} + 25 \cdot \frac{25}{148} + 50 \cdot \frac{50}{148}$

$= 39.28$

and $E[Y] = 40 \cdot \frac{1}{4} + 33 \cdot \frac{1}{4} + 25 \cdot \frac{1}{4} + 50 \cdot \frac{1}{4} = 37$

$E[X]$ is larger since the student is more likely to be selected from a bus carrying a large number of students. When the bus driver is selected the 4 buses are equally likely.

6. Let X = # of complaints ; $X \sim \text{Poisson}(\lambda = 1.2)$

a. $P(X=1) = \frac{e^{-1.2}\, 1.2^1}{1!} = .361$

b. $P(X \geq 1) = \sum_{k=1}^{\infty} \frac{e^{-1.2}\, 1.2^k}{k!} \underset{MTB}{=} .699$

c. $P(X \leq 1) = \frac{e^{-1.2}\, 1.2^0}{0!} + \frac{e^{-1.2}\, 1.2^1}{1!} = .663$

Extra Credit: $P(A \cap (B \cup C)) = P((A \cap B) \cup (A \cap C))$

$\underset{\text{Addition Rule}}{=} P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$

$\underset{\text{indep.}}{=} P(A)P(B) + P(A)P(C) - P(A)\,P(B \cap C)$

$= P(A)\,[P(B) + P(C) - P(B \cap C)]$

$= P(A) \cdot [P(B \cup C)]$ $\therefore$ A and $B \cup C$ are indep.